DETERMINING THE TORQUE TRANSMITTED BY A "DRY-FLUID" COUPLING

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Formulas are derived for calculating the torque transmitted by an electromagnetic "dry-fluid" coupling as a function of the rheological characteristics of the working medium as well as the size and the shape of the active surfaces.

The magnetorheological properties of ferromagnetic dispersions have been utilized in the development of very efficient "dry-fluid" couplings, which offer several advantages over other types. Such couplings are becoming more and more widespread in engineering and industrial practice, typically in metal processing [1]. As yet, however, there is no known method by which the dynamic performance of such couplings can be calculated taking into account the magnetorheological properties of the working medium.

The authors attempt here to determine, to the first approximation, the torque transmitted by a "dry-fluid" coupling with a medium the rheological properties of which in a magnetic field are described by the following defining equation:

$$p_{ik} = -\delta_{ik}p + 2kh_{\cdot}^{n-1}\dot{e}_{ik}. \tag{1}$$

It is well known that, in the phenomenological sense, the rheological properties of many dispersion systems are adequately well described by this equation [2, 3]. The rheological parameters k and n for any system must, of course, depend on the magnetic field intensity. The magnetorheological effect in a dispersion system used as the working medium for a "dry-fluid" coupling consists of reversible changes in these parameters incurred by the application of a magnetic field. In a constant magnetic field both rheological parameters depend on the physical properties of the dispersion, especially of its solid phase.

According to the shape of its active surfaces, a "dry-fluid" coupling can be of the face, the sleeve, or the hybrid type. In face couplings, the torque is transmitted from the driver disk to the follower disk, both coupled mechanically through the working medium. In sleeve couplings, the working medium fills the gap between two concentric cylindrical surfaces. In hybrid couplings, the surfaces of flat disks and of cylindrical sleeves are active.

We will first determine the torque transmitted across a pair of active surfaces (driver and follower) in a face coupling. Let us assume that one disk (the follower) is stationary while the other (the driver) rotates at a certain angular velocity ω (Fig. 1). The gap between both disks is δ. The hermetically sealed space between the disks contains a magnetorheological medium the behavior of which is described by Eq. (1). In cylindrical coordinates r, φ, z (the Oz axis being perpendicular to the planes of the disks), as long as $\delta \ll R_1$, we may assume with sufficient accuracy that $v_r = v_z = 0$ and $v_\varphi = f(r, z)$. The continuity condition with respect to strains will then be satisfied identically.

The differential equations become

$$\frac{\partial p}{\partial r} = \rho_0 \frac{v_\varphi^2}{r}, \quad \frac{\partial p_{\varphi z}}{\partial z} + \frac{2p_{\varphi r}}{r} = 0. \tag{2}$$

The intensity of strain rates is

$$h = \sqrt{\left(\frac{\partial v_\varphi}{\partial r} - \frac{v_\varphi}{r}\right)^2 + \left(\frac{\partial v_\varphi}{\partial z}\right)^2}.$$

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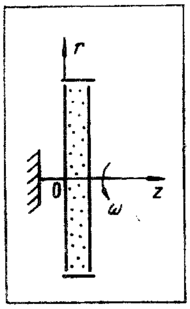

Fig. 1. Schematic representation of a disk coupling.

The first term under the square root will be assumed equal to zero. Considering that within the range $0 \leq z \leq \delta$

$$\partial v_\varphi/\partial z > 0,$$

we have

$$h = \partial v_\varphi/\partial z.$$

Moreover,

$$p_{r\varphi} = 0, \quad p_{\varphi z} = kh^{n-1}\left(\frac{\partial v_\varphi}{\partial z}\right) = k\left(\frac{\partial v_\varphi}{\partial z}\right)^n.$$

Inserting this into (2) yields

$$kn\left(\frac{\partial v_\varphi}{\partial z}\right)^{n-1}\frac{\partial^2 v_\varphi}{\partial z^2} = 0.$$

After integration,

$$v_\varphi = u(r)z + M(r).$$

In the physical sense

$$v_\varphi|_{z=0} = 0 \text{ and } v_\varphi|_{z=\delta} = \omega r,$$

whence

$$M(r) = 0, \quad u(r) = \frac{\omega r}{\delta} \text{ and } v_\varphi = \frac{\omega r}{\delta} z.$$

For the shear stress $P_{\varphi z}$ we obtain the following expression:

$$p_{\varphi z} = k\left(\frac{\omega r}{\delta}\right)^n.$$

It becomes evident now that stress $P_{\varphi z}$ is independent of z, which agrees with the physical conditions of the problem.

The torque transmitted through the working medium is

$$M_{cr} = \int_{R_1}^{R_2} 2\pi r^2 p_{\varphi z} dr = \frac{2\pi k\omega^n}{\delta^n(n+3)}\left(R_2^{n+3} - R_1^{n+3}\right).$$

For this formula we can see that, when $\omega = 0$, the torque transmitted by a coupling is zero, i.e., with a working medium which has no elastic limit it is impossible to transmit torque without producing slip between the disks.

The rheological equation for a medium which obeys a power law but has an elastic limit τ_0 is

$$p_{ik} = -\delta_{ik}p + 2\left(\frac{\tau_0}{h} + kh^{n-1}\right)e_{ik} \tag{3}$$

(generalized Balkley–Herschel equation). Moreover,

$$p_{\varphi z} = \tau_0 + k\left(\frac{\partial v_\varphi}{\partial z}\right)^n$$

and the magnitude of the torque is

$$M_{cr} = \frac{2}{3}\pi\tau_0\left(R_2^3 - R_1^3\right) + \frac{2\pi k\omega^n}{\delta^n(n+3)}\left(R_2^{n+3} - R_1^{n+3}\right).$$

Slip between the disks will occur in this case if the resisting torque at the follower shaft is greater than M_0:

$$M_0 = \frac{2}{3}\tau_0\pi\left(R_2^3 - R_1^3\right).$$

We will now determine the torque transmitted by a sleeve coupling, where motion is transferred from the driver to the follower through a working medium contained between two concentric cylinders. In practice the radial gap between the cylinders is small as compared to the radii of these cylinders. Therefore, we may let

$$v_r = v_z = 0, \quad v_\varphi = f(r).$$

With this, we obtain the following differential equations:

$$\frac{dp}{dr} = \rho_0 \frac{v_\varphi^2}{r},$$

$$\frac{dp_{r\varphi}}{dr} + \frac{2p_{r\varphi}}{r} = 0.$$

The second of these equations yields

$$p_{r\varphi} = C/r^2, \tag{4}$$

where C is the integration constant. From the rheological equation (3) we have

$$p_{r\varphi} = \tau_0 + k\left(\frac{dv_\varphi}{dr} - \frac{v_\varphi}{r}\right)^n. \tag{5}$$

If the outer cylinder is driving and the inner one is driven, then

$$\frac{dv_\varphi}{dr} - \frac{v_\varphi}{r} > 0.$$

In the converse case

$$\frac{dv_\varphi}{dr} - \frac{v_\varphi}{r} < 0$$

and the stress component can be expressed as

$$p_{r\varphi} = -\tau_0 - k\left|\frac{dv_\varphi}{dr} - \frac{v_\varphi}{r}\right|^n.$$

The constant in Eq.(4) is

$$C = M/2\pi l.$$

The maximum torque which a coupling can transmit without slipping is

$$M_{\max} = 2\pi l \tau_0 R_1^2 \qquad (R_1 < R_2).$$

If the outer cylinder is driving and the inner one is driven, then equating (4) and (5) will yield

$$v_\varphi = \frac{r}{k^n} \int_{R_1}^{r} \frac{1}{\nu} \sqrt[n]{\frac{C}{\nu^2} - \tau_0}\, d\nu, \quad R_1 \leqslant r \leqslant R_2.$$

When the torque transmitted by a coupling is greater than M_{max}, then

$$\left.\frac{dv_\varphi}{dr} - \frac{v_\varphi}{r}\right|_{r=R_2} > 0,$$

i.e., slipping is inevitable.

In the case of hybrid couplings, the transmitted torque is the sum of the torques transmitted by the flat disks and by the cylindrical sleeves.

NOTATION

- p is the hydrostatic pressure (isotropic component);
- p_{ik} are the components of the stress tensor;
- k is the consistency index;
- n is the power-law exponent;
- h is the intensity of the strain rates;

$\dot{e}_{ik}$	are the components of the strain tensor;
ρ_0	is the density of the working medium;
τ_0	is the elastic limit in shear;
ω	is the angular velocity of the driver;
v_z, v_φ, v_r	are velocity components;
R	is the disk radius;
R_1, R_2	are the radii of the cylindrical surfaces in a (sleeve) coupling;
δ	is the distance between disks;
l	is the length of the cylindrical surface;
M, M_{max}	are the operating and maximum torque transmitted by the coupling;
$\delta_{ik} = {}^{1}_{0}\Big\vert {}^{i=k}_{i\neq k}$	is the Kronecker delta.

LITERATURE CITED

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